Discrete-time modeling and animation with mechatronics approach for control education

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***Abstract*— A major problem in teaching and learning control theory is acquiring unconventional systems thinking. However, using discrete-time models does not require infinitesimal mathematics, so building on the knowledge acquired at the high school level is possible. This paper aims to show how different discrete-time models (discrete-time convolution, autoregressive moving average model) can be used to illustrate the basic principles of control engineering and describe dynamic systems. With these and similar methods, the focus is shifted to understanding mechatronic engineering, thus increasing the effectiveness of the teaching.**

***Keywords*— discrete-time model; systems engineering; discrete-time convolution; autoregressive moving average model; interactive learning**

I. Introduction

A. General trends in education

The understanding of the principles of control theory has always posed difficulties for engineering students. Although only a few of them could apply these principles professionally, it did not pose a problem because relatively simple analog PID controllers were used in most applications, based on the technological level of the time. Since advanced controllers were rarely used, a small number of experts were sufficient to handle them. However, today, with the availability of high-capacity computers, complex digital regulators can be employed, necessitating a larger number of experts to design and implement them. In order to achieve this goal, the methods of teaching control theory—a field of applied mathematics—must be revised to enhance the efficiency of engineering education.

Today, due to technological demands, the development of engineering education is the focus of many researchers. The role of immediate feedback in improving efficiency in teaching engineering mathematics is discussed in [1]. Vieira et al. present a teaching process for human-computer interaction, combining traditional teaching methods with project-based learning and active learning, using interactive learning materials [2]. More interactive features can be found [3], where they use a 3-D interactive learning environment for control engineering education. The following article uses a computer-aided animation tool [4]. More about interactive education can be found [5] [6] [7] [8] [9]. Another interesting topic is online laboratories. Vergara et al. write about post-Covid educational trends [10]. An online laboratory was used before the pandemic [11] [12]. Other related studies can be found [13] [14] [15]. One more topic is distance learning, where the impacts and challenges can be found in the following articles [16] [17].

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B. The simplified goal of the classical control theory

In preparation for an Erasmus project, we asked Hungarian, Slovenian, Italian, and Croatian students how difficult they found the knowledge of control engineering compared to other subjects. This research was published in [9]. It turned out that most of them find this knowledge more difficult than other subjects (Fig. 1). This may be partly because the mathematical and physical foundations of most engineering subjects (such as mechanics, thermodynamics, optics, electronics, and fluid mechanics) are already learned at primary and high school level. Although university education quickly moves beyond the high school level of knowledge and developing an appropriate engineering mindset is a long process, building on the mathematical and physical mindset developed over many years in these areas is possible.

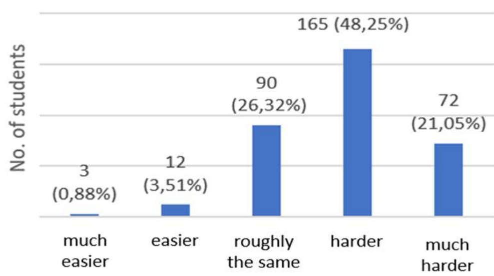


Fig. 1: Relative difficulty of control subjects compared to other subjects according to engineering students [9]

A similar conclusion was reached by the IFAC WC 2017 education roundtable discussion: feedback is the most important operation and saleable product of control theory and the biggest challenge to its teaching (Fig. 2). In contrast, the basics of control theory are currently not taught at all in high school, and engineering students have to deal with basic attitudinal deficits, which is why this subject is considered difficult. Understanding feedback is also an important part of everyday thinking; understanding its role would be important for everyone, and learning the basics does not require a strong knowledge of mathematics, so there is no reason why young people should not learn about this topic.

The role of laboratory measurements is crucial in engineering and other applied science courses, while the availability of instruments is limited due to time constraints and funding difficulties. The rapid development of computing and the internet has enabled remote measurements via the internet, and the need to increase efficiency has made their widespread use inevitable. Virtual laboratories are available, which place experimentation in a simulated environment [18] [19].

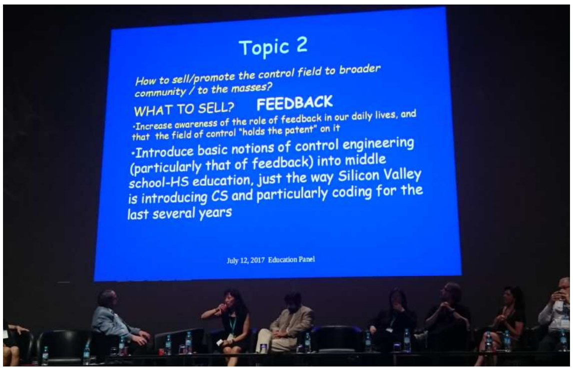


Fig. 2. Education Round Table, IFAC World Congress 2017

C. *Structure of the paper*

The next Chapter describes the mathematical background of classical basic continuous time controller design, which we want to teach in discretet time first. In Chapter III, we present a thought experiment that implies the impulse response of a discrete-time system demonstrated in the form of animation. In practice, students can make observations (measurements) using a virtual bicycle model. With its generalization, we arrive at discrete-time convolution. Here, we recognize that we must use infinite series, which we try to write in closed form. Chapter IV compares the continuous and discrete time transfer functions. The continuous time transfer function is based on Laplace transformation and time shifting operator is introduced for the discrete time. In this paper, we do not deal with controller design, but we present the discrete-time version of the control theory toolset to use as a basis later when teaching advanced control theory. Chapter V concludes the paper.

II. Problem Statement (Convolution Integral)

The basic problem of classical control theory is to write down the Laplace operator transfer function of both the controlled plant $W_p(s)$ and the controller $W_c(s)$ and create the closed control circuit shown in Fig. 3, where $U_r(s)$, $Y(s)$, $E(s)$, and $U(s)$ are the Laplace transformed reference signal, output signal, error signal, and control signal, respectively.

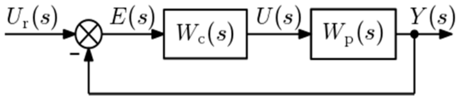


Fig. 3. Classical closed control loop

In the case of continuous-time systems, it is common to use a closed form for the transfer function. After providing a brief overview of this case, we will introduce the analogous tools for discrete-time systems in Chapter IV. Generally, we are dealing with systems that can be described n-th order linear differential equations of form

$$\sum_{i=0}^{n} a_i \cdot y^{(i)}(t) = \sum_{i=0}^{r} b_i \cdot u^{(i)}(t), \qquad (1)$$

where $u(t)$ is the input of the system and $y(t)$ is the output.

Here, without going into the mathematical details, we introduce the most important concepts that become important in the later learning of control theory. In continuous time, the output (the solution of the differential equations) can be generated by the so-called convolution integral

$$y(t) = \int_{-0}^{t} w(t-\tau) \cdot u(\tau) d\tau \quad t \geq 0, \qquad (2)$$

where $w(t)$ is the impulse response function. The use of -0 as a lower bound emphasizes that the history of the system should be taken into account with an appropriate left limit of the initial value at $t = 0$.

If we transfer from the time domain to the Laplace domain with the help of the Laplace transform, then (2) the convolution integral is simplified to an ordinary multiplication

$$Y(s) = W(s) \cdot U(s) \qquad (3)$$

where $Y(s)$, $U(s)$ and $W(s)$ are the Laplace transforms of the system's output, input, and impulse response functions. Equation (3) provides a closed formula for the output and shows the importance of the impulse response function as well as the Laplace transform. Since the differential operator in the case of the Laplace transformation is an s multiplier, the transfer function can be written directly from (1)

$$W(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{i=0}^{r} b_i \cdot s^{-i}}{\sum_{i=0}^{n} a_i \cdot s^{-i}} \qquad (4)$$

When designing the controller, we move from the Laplace operator domain to the complex frequency domain and design the complex frequency [20] [21] transfer function $W_x(j\omega)$ of the open loop so that the closed loop remains stable.

$$W_x(j\omega) = W_c(j\omega) \cdot W_p(j\omega) \qquad (5)$$

During planning, of course, we can only change the transfer function $W_c(j\omega)$ of the controller.

According to the general experience of control theory instructors and the survey mentioned in the introduction, thinking in frequency or Laplace domain as well as understanding the essence of convolution integral, which justifies the use of the transfer function, requires students to have an abstraction. It causes significant problem for most students if we use the traditional teaching methods, and do not provide them a special way of thinking. That's why we decided to transform our education to create a synergy of subjects and, here specifically, to subordinate mathematics education to engineering thinking.

Our educational hypothesis is the following: on the one hand, students understand phenomena and manipulations better in the time domain than in the frequency domain (not to mention the Laplace operator domain), on the other hand, the most difficult abstraction for them is to change from finite time to infinitely short time and to sum up (integrate) the infinitesimally small effects of events that take place in infinitely short time.

As an educational innovation, we propose taking advantage of the closed control loop shown in Fig. 3, which can be represented by the impulse transfer functions obtained through the Z transform of sampling systems. The novelty lies in the fact that we introduce the pulse transfer function at the beginning of university studies using high school mathematics. To achieve this, we do not need the Laplace domain; instead, we utilize a simple time-shifting operator in the time domain. Although we do not mention the Z transform in this phase, we use the notation z to represent the time-shifting operator, indicating the connection between them. We assert that understanding the convolutional summation in discrete time is much easier than comprehending the convolutional integral, which can also assist students in grasping the control theory of continuous-time systems at a later stage.

III. Use of Discrete-Time Models in Preparation for Studying Classical Control Theory

A. *Measurements through a virtual system (animation) to demonstrate discrete-time impulse response and convolution*

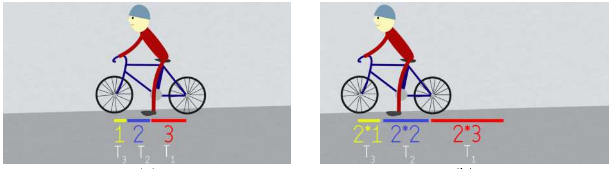
An example is described in detail where, due to the nature of the system, it is relatively easy to imagine that the input signal is a series of impulses.

Suppose that a cyclist can push down on the pedal very quickly and thus imposes a given magnitude of impulse on the system consisting of the cyclist and his bicycle. Using measurements, form a discrete-time model where the discrete-time input signal $u[k]$ is the sequence of impulses from pedal depression, and $y[k]$ is the output signal, which is also a sequence of certain values.

The output signal $y[k]$ can be various, for example, the velocity at each discrete time or the total distance traveled from the start. We will see that discussing these two cases easily introduces the so-called Finite and Infinite Impulse Response systems. In the following, we use subscript 'v' when discussing the 'velocity response' of the system and subscript 'd' when studying the 'distance response.' For the sake of simplicity, we use unit time intervals as time steps and suppose SI units in our calculations and do not write them.

1) *Measurement: Impulse response*

Let us examine the discrete-time response of the system to a unit impulse. If the cyclist presses the pedal once at time $t[0]$ and a unit impulse is generated, then in our example, we find that the velocity of the cyclist at time instants $t[0], t[1]$, and $t[2]$ is $y_v[0] = 3, y_v[1] = 2, y_v[2] = 1$, respectively, then (at $t[3]$) the bicycle stops. Furthermore, the covered distance during the first three units of time is 3, 2, and 1 unit; that is, the total covered distance is $y_d[0] = 3, y_d[1] = 5$, and $y_d[2] = 6$, respectively (Fig. 4 (a)).



(a) (b)

Fig. 4: a) Effect of a unit impulse b) Effect of a pulse with the magnitude of two units

Based on the measurement, the discrete-time velocity response $w_v[k]$ of the system to the unit pedal impulse is

$$\begin{aligned} y_v[0] &= w_v[0] = 3, \\ y_v[1] &= w_v[1] = 2, \\ y_v[2] &= w_v[2] = 1 \\ y_v[k] &= w_v[k] = 0, \text{ if } k > 2 \end{aligned} \tag{6}$$

Based on the measurement, the discrete-time distance response $w_d[k]$ of the system to the unit pedal impulse is

$$\begin{aligned} y_d[0] &= w_d[0] = 3, \\ y_d[1] &= w_d[1] = 5, \\ y_d[2] &= w_d[2] = 6 \\ y_d[k] &= w_d[k] = 6, \text{ if } k > 2 \end{aligned} \tag{7}$$

Suppose there exists $K \in \mathbb{Z}$ for which $w[k] = 0$ if $k > K$, then the system is called Finite Impulse Response (FIR) system. Otherwise, the system is called Infinite Impulse Response (IIR) system. Based on (6) and (7), our system is a FIR system if we consider the velocity as the output, while it is an IIR system if the covered distance is considered to be the output.

2) *Measurement: Linearity of the system*

In our example, we are dealing with the motion of a bicycle, and we want to introduce some control theory tools within a simplified linear model.

A real system can only be linear in a specific operating range and within a certain accuracy constraint, so the assumption of linearity usually implies a significant simplification of the system. In the bicycle example, it is enough to think that pushing the pedal very hard can break. But the cause of a bicycle slowing down is also complex, caused by air and rolling resistance and the friction of the components, of which several types are present at the same time. Most of these effects result in the nonlinearity of the system. For example, only the so-called viscous friction can be considered linear, which is experienced when the bearings of the bicycle are properly lubricated, and the surfaces of the connecting parts do not slide on each other but on the oil layer between them. Additionally, the degree of different deceleration effects also depends on the speed.

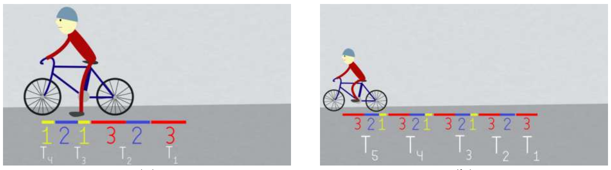
Since we want to teach an analysis toolset of linear systems, for didactical purposes, we use a virtual bicycle prepared to be linear in our example. Students can do measurements with this system. Generally, we would need to perform many measurements to 'prove' the linearity of a system. In this exercise students are not required to 'prove' the linearity, they have to do only one measurement to 'check' the linearity as follows. Suppose you press the pedal twice the force as in the first measurement. In this case, because of the linearity, we find that the distance traveled per unit of time also doubles, and after the third unit of time, the bicycle is no longer moving (Fig. 4 (b)).

3) Measurement: Effect of two unit impulses

As a first step to generalization, let us examine the effect of two-unit impulses with a unit time lag.

Velocity response: the velocity of the cyclist at $t[0]$ is 3 due to the first pedal push, $y_v[0] = 1 \cdot 3$, at $t[1]$ the velocity is the sum of 2 due to the first pedal push and 3 due to the second pedal push, $y_v[1] = 1 \cdot 2 + 1 \cdot 3 = 5$, at $t[2]$ the velocity is the sum of 1 due to the first pedal push and 2 due to the second pedal push, $y_v[2] = 1 \cdot 2 + 1 \cdot 1 = 3$, at $t[3]$ the velocity is 1 due to the second pedal push, $y_v[3] = 1 \cdot 1 = 1$, then the bicycle stops, that is, $y_v[k] = 0, \text{if } k > 3$.

Distance response: till the end of the first unit of time, the bicycle covers a distance of $y_d[0] = 1 \cdot 3$ due to the first pedal push, till the end of the second unit of time, it covers a distance of $y_d[1] = 1 \cdot 5 + 1 \cdot 3 = 8$ units due to the first and second pedal push, till the end of the third time unit it covers a distance of $y_d[2] = 1 \cdot 6 + 1 \cdot 5 = 11$ units, till the end of the fourth time unit it covers a distance of $y_d[3] = 1 \cdot 6 + 1 \cdot 6 = 12$ units, then the covered distance is a constant, $y_d[k] = 12, k > 3$ (Fig. 5 (a)).



(a) (b)

Fig. 5: a) Effect of two unit pulses b) Effect of the discrete-time unit step function

4) Measurement: Effect of the discrete-time unit step function

If the cyclist presses the pedal with a unit of impulse per unit of time, the effect of these impulses should be summed over time as shown above. Velocity response: the velocity of the cyclist at $t[0]$ is 3 due to the first pedal push, $y_v[0] = 1 \cdot 3$, at $t[1]$ the velocity is the sum of 2 due to the first pedal push and 3 due to the second pedal push, $y_v[1] = 1 \cdot 2 + 1 \cdot 3 = 5$, at $t[2]$ the velocity is the sum of 1 due to the first pedal push, 2 due to the second pedal push, and 3 due to the third pedal push, $y_v[2] = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 = 6$, at $t[3]$ the velocity is the sum of 1 due to the second pedal push, 2 due to the third pedal push and 3 due to the fourth pedal push, $y_v[3] = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 = 6$, then the bicycle travels at a constant velocity of $y_v[k] = 6, \text{if } k > 3$.

Distance response: till the end of the first unit of time, the bicycle covers a distance of $y_d[0] = 1 \cdot 3$ due to the first pedal push, till the end of the second unit of time, it covers a distance of $y_d[1] = 1 \cdot 5 + 1 \cdot 3 = 8$ units due to the first and second pedal push, till the end of the third time unit it covers a distance of $y_d[2] = 1 \cdot 6 + 1 \cdot 5 + 1 \cdot 3 = 14$ units due to the first, second, and third pedal push, till the end of the fourth time unit it covers a distance of $y_d[3] = 1 \cdot 6 + 1 \cdot 6 + 1 \cdot 5 + 1 \cdot 3 = 20$ units, then the covered distance is a constant, etc. (Fig. 5 (b)).

5) Measurement: Response to an arbitrary input

If the cyclist does not act on the system with the same impulse per unit of time, then the effects proportional to the magnitude of the impulse should be summed up with time delay. For example, if one leg of the cyclist is stronger and therefore, every second impulse is twice as large, i.e.,

$$u[0] = 1, u[1] = 2, u[2] = 1, u[3] = 2, \ldots$$

then the velocity response is

$$y_v[0] = 1 \cdot 3,$$
$$y_v[1] = 1 \cdot 2 + 2 \cdot 3 = 7,$$
$$y_v[2] = 1 \cdot 1 + 2 \cdot 2 + 1 \cdot 3 = 8,$$
$$y_v[3] = 2 \cdot 1 + 1 \cdot 2 + 2 \cdot 3 = 9,$$

then velocity values 8 and 9 are alternating. The virtual bicycle measurements discussed above can be studied using the following link (available only in Hungarian): https://www.youtube.com/watch?v=alFZRrWkoLw

B. Generalization of the animation

To formalize the ideas above, let us introduce the function

$$\delta[k] = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases}, \quad k \in \mathbb{Z}, \tag{8}$$

which is the representation of the unit impulse at 0. Then $\delta[k-i], k \in \mathbb{Z}$ is the representation of the unit impulse at $i \in \mathbb{Z}$. The generalization of Fig. 4 (a) is shown in Fig. 6, where $w[k]$ is the system response for unit impulse, $\delta[k]$.

The generalization of the measurement 5 is shown in Fig. 7. Using δ, sequence $u[i] \cdot \delta[K-i], i \in \mathbb{Z}$ represents the input impulse of magnitude $u[K]$ at fixed K, and for sequence $u[K]$ we have decomposition

$$u[K] = \sum_{i=-\infty}^{\infty} u[i] \cdot \delta[K-i], \quad K \in \mathbb{Z} \tag{9}$$

The response of the system to input $\delta[k]$ is the impulse response $w[k]$ (Fig. 6). Therefore, the response to $u[i] \cdot \delta[k-i], i \in \mathbb{Z}$ is $u[i] \cdot w[k-i], i \in \mathbb{Z}$, furthermore, because of the linearity, the system output for input $u[k], k \in \mathbb{Z}$ is

$$y[k] = \sum_{i=-\infty}^{\infty} u[i] \cdot w[k-i], \quad k \in \mathbb{Z} \tag{10}$$

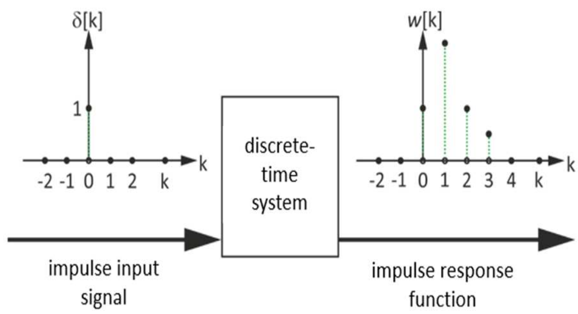


Fig. 6: Discrete-time impulse response

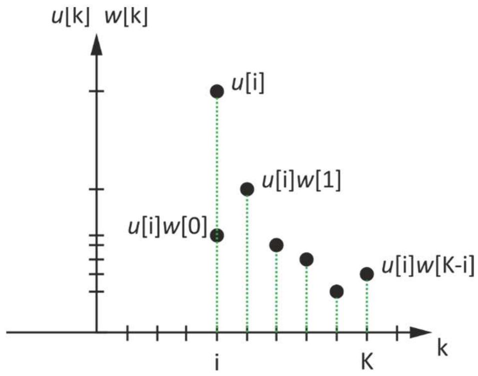


Fig. 7: Calculating the output signal with discrete time convolution

If $u[i] = 0$ for all $i < 0$ and $w[i] = 0$ for all $i < 0$ (causality), then (10) is equivalent to a finite sum

$$y[k] = \sum_{i=0}^{k} u[i] \cdot w[k-i], \;\; k \geq 0 \qquad (11)$$

We got the same formula we have in (11), which is called discrete-time convolution of discrete-time functions $u[k]$ and $w[k]$. According to (11), the value of the output signal at a time instant is generated as a linear combination of the input signal values acting on the system in the past. There are no state variables in the expression, the output signal $y[k]$ is determined by the function $w[k]$ and the input signal $u[k]$.

To illustrate the above, consider the following bicycle model. Let the velocity impulse response of the system be

$$w_v[0] = 1, w_v[1] = 3, w_v[2] = 2 \text{ and } \quad w_v[k] = 0 \text{ if } k > 3 \qquad (12)$$

(FIR system). Calculate the velocity response $y_v[k]$ and the displacement response $y_d[k]$ of the system to the discrete-time unit step function. (The unit step function can also be considered as a series of discrete-time unit pulses.)

It is clear that the (total) displacement impulse response of the system is

$$w_d[0] = 1, w_d[1] = 4, \text{ and } \quad w_d[k] = 6 \text{ if } k > 1 \qquad (13)$$

Calculate values $y_v[k]$ and $y_d[k]$, $k = 0,1, \dots,5$.

Rows of Table I and Table II contain response functions $u[i] \cdot w_v[k-i]$ and $u[i] \cdot w_d[k-i]$, respectively while in the last rows, we have the sums of these functions that provide the values of the outputs $y_v[k]$ and $y_d[k]$, respectively.

I. TABLE: CALCULATION OF THE VELOCITY OUTPUT AS THE CONVOLUTION OF INPUT AND THE IMPULSE RESPONSE

k	-1	0	1	2	3	4	5
$u[k]$	0	1	1	1	1	1	1
$u[0] \cdot w_v[k]$	0	3	2	1	0	0	0
$u[1] \cdot w_v[k-1]$	0	0	3	2	1	0	0
$u[2] \cdot w_v[k-2]$	0	0	0	3	2	1	0
$u[3] \cdot w_v[k-3]$	0	0	0	0	3	2	1
$u[4] \cdot w_v[k-4]$	0	0	0	0	0	3	2
$u[5] \cdot w_v[k-5]$	0	0	0	0	0	0	3
$y_v[k]$	0	3	5	6	6	6	6

It is generally beneficial to express the impulse response in closed form. While obtaining the values of the impulse response through measurements is generally impossible, it becomes feasible if the differential equation of the system is available.

II. TABLE: CALCULATION OF THE DISPLACEMENT OUTPUT AS THE CONVOLUTION OF INPUT AND THE IMPULSE RESPONSE

k	-1	0	1	2	3	4	5
$u[k]$	0	1	1	1	1	1	1
$u[0] \cdot w_d[k]$	0	3	5	6	6	6	6
$u[1] \cdot w_d[k-1]$	0	0	3	5	6	6	6
$u[2] \cdot w_d[k-2]$	0	0	0	3	5	6	6
$u[3] \cdot w_d[k-3]$	0	0	0	0	3	5	6
$u[4] \cdot w_d[k-4]$	0	0	0	0	0	3	5
$u[5] \cdot w_d[k-5]$	0	0	0	0	0	0	3
$y_d[k]$	0	3	8	14	20	26	32

IV. DISCRETE-TIME TRANSFER FUNCTION FOR CLOSED FORM IMPULSE RESPONSE

Our goal is to create an analog toolset for studying discrete-time systems. Instead of the Laplace transform, we use the so-called time-shifting operator.

In the classical structure of control theory education, the z transform is introduced after the Laplace transform on its basis. Since the theory of integral transforms is too complex for first-year students, we need more straightforward tools to deal with the basics of control theory at the beginning of the training. For this purpose, we introduce the time-shifting operator and discrete-time convolution instead of Laplace transform and continuous-time convolution. Introducing time-shifting operators and discrete-time convolution requires only high school-level mathematics knowledge.

The differential equation (1) can be transformed into the form

$$\sum_{i=0}^{n} a_{di} \cdot y[k-i] = \sum_{i=0}^{r} b_{di} \cdot u[k-i] \qquad (14)$$

This form can be called ARMA-type formalization: the moving average part is a linear combination of the input signal's current and some past values, while the autoregressive part is a linear combination of the output signal's current and some past values. The value of the output signal is the difference between the moving average part and the autoregressive part. Establish the ARMA-type equation of a real system only needs the determination of coefficients a_{di} and b_{di}.

Let us introduce time shifting operator z by

$$x[k+1] = zx[k], \;\; i \in \mathbb{Z} \qquad (15)$$

where $x[k], k \in \mathbb{Z}$ is a discrete signal. Formally, a signal can be shifted in time by a unit with multiplication by z. The operator z^i represents the application of the time shifting operator i times, that is

$$x[k+i] = z^i x[k], \;\; i \in \mathbb{Z} \qquad (16)$$

With the use of the time-shifting operator, (14) has the form

$$\sum_{i=0}^{n} a_{di} \cdot z^{-i} y[k] = \sum_{i=0}^{r} b_{di} \cdot z^{-i} u[k] \quad (17)$$

Based on this equation, the so-called discrete transfer function can be expressed as

$$W(z) = \frac{Y(z)}{U(z)} = \frac{\sum_{i=0}^{r} b_{di} \cdot z^{-i}}{\sum_{i=0}^{n} a_{di} \cdot z^{-i}} \quad (18)$$

$$Y(z) = W(z)U(z) \quad (19)$$

(18) and (19) have the same role in discrete-time systems as (4) and (3) in continuous-time systems. Based on Fig. 6 and Fig. 7, the physical content of (18) is easier to understand than (3). Anyway, we cannot physically imagine the domain of the Laplace operator in general.

V. Conclusion

Although both the description of control theory and the practical design are based on manipulations in the frequency domain and the Laplace domain, the introduction of the time-shifting operator and discrete-time convolution allows the basics of control theory to be discussed from the beginning of engineering education, building only on high school mathematics. This teaching method helps to close the gap between studying the theoretical foundations and the applications in engineering, which causes a lack of interest in learning mathematics at the beginning of the training and a lack of mathematical knowledge and difficulty in applying it later on. The teaching method presented relies on the active participation of the students. On the one hand, the study of the mathematical tools of discrete-time systems can be easily achieved through self-study and homework, and on the other hand, measurements and observations are made from the outset on a virtual system that can be modified as required.The basic concepts of control theory can thus be discussed before higher-level analysis is taught, and when the difficult definitions and abstract formulas appear, reference can be made to formulas previously written and understood for discrete-time systems, which look similar. Most students have trouble understanding abstract mathematics, so abstract control theory is, for the majority, just formulas to be learned; deeper understanding is lost. The study and understanding of discrete-time systems alone can contribute significantly to the understanding of the description and modeling of dynamic systems.We have been assigning the same task in the control engineering subject for many years. Our experience is that since we started teaching the discrete-time convolution described in the paper to first-year students, the average length of students' submissions has gradually increased because the students intend to provide a deeper and, therefore, more extended analysis of the results.

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